Audience. In all of the writing you do for a course based on this text, take your audience to be your fellow classmates. That will determine the amount of detail you need to include—do not skip important details, and do not include details that are not essential.

Sentences! The most important rule is that your writing should consist solely of complete sentences. A sentence starts with a capital letter and ends with a period. If you have a long calculation, you might want to neatly display it, but it should still be part of a sentence; something like this:

> Our result then follows from a calculation:
>
> $$\begin{aligned} \text{blah} &= \text{blah} \\ &= \text{blah} \\ &\vdots \\ &= \text{blah}. \end{aligned}$$

Use of symbols. For better readability, do not start a sentence with a mathematical symbol (i.e., a mathematical symbol does not count as a capital letter):

No: f is injective by not surjective
Yes: The function f is injective by not surjective.

In informal mathematical writing—the kind you might use on a blackboard or on scratch paper—it is common to use the symbols shown below:

$\Rightarrow$: "implies"
$\Leftrightarrow$: "if and only if"
$\forall$: "for all"
$\exists$: "there exists"

In your formal written work, e.g., homework assignments, do not use these. Instead, write out the words. It is slightly more work for you, but it makes life easier for your reader. Of course, you will need to use symbols in your writing—just do not use those listed in formal writing. On the other hand, here are some symbols from logic we encourage you to never use, even in informal work: "$\wedge$" for "and", "$\vee$" for "or", and "$\sim$" for "not". Again, it is easier on your reader to use the word instead of these symbols.

Red herrings. Whenever you a finish a proof by contradiction and now have the ideas in front of you, ask yourself whether a straightforward proof (without contradiction) is at least as clear as the one you have given. If so, make the change. It makes for a less convoluted argument. Apply similar reasoning to any proof where you have replaced the statement of the result by its (logically equivalent) contrapositive (not Q implies P, rather than P implies Q).

More generally, review your proofs to see if you have included irrelevant information.

The "backwards" proof. Theorem. Suppose $x \in \mathbb{R}$. Then $(x+1)^2 - (x-1)^2 = 4x$.

Incorrect proof. Calculate:

$$\begin{aligned} (x+1)^2 - (x-1)^2 &= 4x \\ (x^2+2x+1) - (x^2-2x+1) &= 4x \\ x^2+2x+1-x^2+2x-1 &= 4x \\ 4x &= 4x. \end{aligned}$$

□

The problem with this "proof" is in its first line: it seems to assert as true exactly what it is trying to prove—circular reasoning. To make the mistake even more clear, consider the following false statement, which uses the same reasoning:

Theorem. In $\mathbb{R}$, we have

$$1 = 0.$$

Incorrect proof. Calculate:

$$\begin{aligned} 1 &= 0 \\ 0 \cdot 1 &= 0 \cdot 0 \\ 0 &= 0. \end{aligned}$$

□

To fix the proof of the original theorem, one could just list the lines of the proof in reverse order—hence, the moniker "backwards"—starting with $4x = 4x$. However, notice another flaw with the proof: by just listing these lines, we break the rule that a proof consists of sentences. Here is the correct form, fixing both problems:

Theorem. Suppose $x \in \mathbb{R}$. Then $(x+1)^2 - (x-1)^2 = 4x$.

Proof. Calculate:

$$\begin{aligned} (x+1)^2 - (x-1)^2 &= (x^2+2x+1) - (x^2-2x+1) \\ &= x^2+2x+1-x^2+2x-1 \\ &= 4x. \end{aligned}$$

□

Miscellaneous.

» To prove a statement is false, give a specific concrete counterexample. Try to find the simplest one. The counterexample is more convincing and easier on the reader than an abstract argument. Conversely, to prove a statement is true, an example, although sometimes helpful, is not a proof. You must show why the statement is true in all instances. Some examples:

Statement: $f(n) = n + 2$ is even for all $n \in \mathbb{Z}$.
Disproof: Note that $f(1) = 3$, which is not even.

Statement: $f(n) = n + 2$ is divisible by 7 for all $n \in \mathbb{Z}$.
False proof: We have $f(12) = 14$, which is divisible by 7.

» If you use the phrase "by definition" in your writing, make sure to be specific: by definition of what? For example, you might write "by definition of a Hausdorff space". Using the phrase "by definition" in isolation is usually ambiguous, and if your reader cannot determine which definition your are referring to, then it is no help at all—everything in mathematics follows from the definitions!

» When writing down a calculation, avoid crossing out terms (for example, when terms cancel in fractions or when they add up to zero). This type of bookkeeping is easy for the writer, who is crossing out sequentially, but is usually confusing for the reader, who sees all of the crosses at once.

TEX pointers.

» When defining a function, use `\colon` rather than `:`, as in

`f\colon X\to Y`.

The latter symbol is regarded as an operator and is padded by unwanted spaces.

» Use the TEX versions of the trig functions, e.g., `\cos(t), \tan(t)`, etc., rather then plain `cos(t), tan(t)`, etc. The same goes for logarithms:`\log(t), \ln(t)`.

» Add a little space before a differential, e.g., `\int x\,dx`, which gives $\int x\,dx$ rather than `\int xdx`, which gives $\int xdx$.

Here is a link to a nice three-page article on mathematical writing by Francis Su:

Some Guidelines for Good Mathematical Writing

and a one-page summary:

Writing Mathematics Well