MATH 112 REVIEW FOR FINAL EXAM

There will be an in-class, closed book/notes/internet final exam 1–4 pm on Wednesday, May 8, in Eliot 314. The following is a precise description of what you are expected to know for the exam.

1. What is a *relation* on a set? An *equivalence relation*?
2. There will probably be an *induction* proof of some sort on the exam.
3. Know the meaning of the following terms used when talking about functions: *domain*, *codomain*, *image (range)*, *surjective*, *injective*, *bijective*, *composition*, *inverse*.
4. What is a *field*?
5. What is $\mathbb{Z}/n\mathbb{Z}$? Is it a field?
6. What are the *order* axioms?
7. What is the *completeness axiom* for the real numbers?
8. Be able to define the following terms associated with sets of real numbers: *upper bound*, *lower bound*, *sup*, *inf*, *max*, and *min*.
9. What are the *complex numbers*? Be able to add, multiply, and divide complex numbers, expressing your result in the form $a + bi$.
10. What is the *polar form* of a complex number? What is the *argument* and *modulus*? Be able to convert between two forms of a complex number. Know what happens to the argument and modulus when you multiply two complex numbers.
11. What is an *open ball* in $\mathbb{R}$ or $\mathbb{C}$? What is an *open set* in $\mathbb{R}$ or $\mathbb{C}$?
12. What is a *closed set* in $\mathbb{R}$ or $\mathbb{C}$?
13. For a real or complex sequence $\{a_n\}$, what does it mean to say $\lim_{n\to\infty} a_n = a$?
14. Be able to give an ε-N proof for the limit of a simple sequence.
15. What is a *monotone sequence*?
16. Be able to prove that *every convergent sequence is bounded.*
17. State and prove the *monotone convergence theorem.*
18. Be able prove that if $\{a_n\}$ and $\{b_n\}$ are convergent sequences, then so is $\{a_n + b_n\}$, and $\lim(a_n + b_n) = \lim a_n + \lim b_n$. (This is an $\varepsilon/2$ proof.)
19. What does it mean for a series to be *absolutely convergent*? *Conditionally convergent*?

20. Know the *tests for convergence* of series and how to use them. A summary of the tests and the solutions to the practice problems on sequences and series appears here: series tests.

21. Know how to sum a *geometric series* (being careful where the series starts).

22. For a real or complex function f, what does it mean to say $\lim_{z\to a} f(z) = L$?

23. Be able to give an ε-δ proof for the limit of a simple function.

24. What does it mean to say a function is *continuous* at a point?

25. What is the definition of the *derivative* of a function at a point?

26. Know the two big theorems about powers series: the first having to do with the existence of a *radius of convergence*, and the second having to do with differentiation.

27. Be able to use the ratio test to find the radius of convergence of a power series.

28. Know the Taylor series for the exponential (denoted $\exp(z)$ or e^z), the sine, and cosine functions. From these, be able to show for all $z \in \mathbb{C}$ that
    - $\exp'(z) = -\exp(z)$, $\cos'(z) = -\sin(z)$, and $\sin'(z) = \cos(z)$;
    - $\exp(iz) = \cos(z) + i\sin(z)$.

29. Be able to compute the *Taylor polynomial* of a given order or the *Taylor series* for a function.