Math 112 Homework, Week 11

As usual, show your work and use sentences in order to receive credit.

PROBLEM 1. Use the comparison theorem (not the limit comparison theorem) to determine whether the following series converge. You may use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$. (Show your work, as always.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^3 + n^2 + 5}$ (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\sqrt{n^2 - 5}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n7^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n-1}\right)$ (e) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - e^{-n}}{5^n + e^{-n}}$

PROBLEM 2. What does the n-th term test say about the convergence or divergence of the following series?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2 + 5n - 7}{2n^2 + 3n - 6}$ (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{\sqrt{n^2 - 5}}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{\ln(n)}$.

PROBLEM 3. Sum the series:

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)}.$$

Hint: partial fractions.

PROBLEM 4. Let $\{a_n\}$ be a sequence of nonnegative real numbers. Prove that $\sum_{n=1}^{\infty} a_n$ converges if and only if its sequence of partial sums $\{s_n\}$ is bounded. (We have done this problem, but writing it up on your own will help in understanding the key role the monotone convergence theorem plays in series tests. Also: note that this is an "if and only if" proof. So you need to prove both implications.)

Note: In the following, you may use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$. As always, *use sentences to explain your reasoning.*

PROBLEM 5. Use *the limit comparison test* to determine whether the following series converge. (Use limit comparison even if you know another method.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)(n+3}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{3n^2 + 2n + 9}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n3^n + 2}$.

PROBLEM 6. Are the following series absolutely convergent, conditionally convergent, or divergent?

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{n^3+2}$ (c) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{e^n}{n^2}.$

PROBLEM 7. Apply the ratio test to each of the following series, and state what conclusion may be drawn:

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n2^n}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^n(n+1)!}$ (d) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n n!}{(2n)!}.$

PROBLEM 8. Apply the integral test to each of the following series, and state what conclusion may be drawn:

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{e^n}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{3n^2+2}{n^3+2n+1}.$

PROBLEM 9.

(a) Why doesn't the integral test directly apply to determine convergence or divergence of the series

$$\sum_{n=1}^{\infty}\frac{\cos(n)}{n^3}\ ?$$

(b) What does the p-series test say about the convergence or divergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{n^{1+e^{-n}}}\ ?$$