Math 112 Homework, Friday Week 9

PROBLEM 1. Find the limit of the following sequence, and provide an ε-N proof:

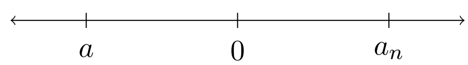
$$\left\{\frac{3n^4+6n^2+1}{n^4+2n^2+3}\right\}.$$

PROBLEM 2. Find the limit of the following sequence, and provide an ε-N proof:

$$\left\{\frac{(-1)^n}{n^2}\right\}.$$

PROBLEM 3. Suppose $\{a_n\}$ is a sequence of positive real numbers and that $\lim_{n\to\infty} a_n = a$.

(a) Use an ε-N argument to prove that $a \geq 0$. Hint: suppose that $a < 0$ and argue that this would imply $a_n < 0$ for some n. The picture below might help:



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What should we take ε to be in order to force a_n to be be negative for large n (in contradiction to the fact that $a_n > 0$ is positive for all n)?

(b) Is it necessarily true that $a > 0$? (Give a proof or explicit counterexample.)

PROBLEM 4. Let $\{a_n\}$ be a sequence of complex numbers, and consider the sequence $\{|a_n|\}$ of real numbers.

(a) Suppose $\lim_{n\to\infty} a_n = a$. Prove that $\lim_{n\to\infty} |a_n| = |a|$ using an ε-N argument. (The reverse-triangle inequality may be of use.)

(b) Give an example for which $\lim_{n\to\infty} |a_n|$ exists but $\lim_{n\to\infty} a_n$ does not.

PROBLEM 5. Prove that $\lim_{n\to\infty} \frac{1}{n^2} \neq 3$ straight from the definition of the limit, i.e., using an ε-N argument. (You will need to find a specific $\varepsilon > 0$ for which there is no response N.)

PROBLEM 6. Find the limit of the following sequences, proving your results using our "new-from-old" limit theorem (not ε-N proofs). Use enough steps so that we can clearly see how that theorem is being used.

(a)

$$\left\{\frac{4n^2+5}{8n^2-2n+3}\right\}$$

(b)

$$\left\{\frac{n}{n^2+3}\right\}.$$