Math 112 Homework, Friday Week 6

PROBLEM 1. Fill in the following table, writing DNE if the quantity does not exist:

	sup	max	inf	min
$(-4, \pi)$				
$[6, 12)$				
$(-\infty, 3]$				
$\{1 + (-1)^n : n \in \mathbb{N}\}$				
$\{\sin(1/x) : x \in \mathbb{R}_{>0}\}$				

.

No explanation is required.

PROBLEM 2. Let S be a subset of an ordered field F, and suppose $\sup(S)$ exists. Fix $c \in F$, and define the set

$$c + S := \{c + s : s \in S\}.$$

Prove that $\sup(c + S) = c + \sup(S)$ by filling in the blanks in the template below. (Your write-up should include the full proof.)

Proof. We first prove that $c+\sup(S)$ is an upper bound for $c+S$. Let $x \in c+S$. The $x = c+s$ for some $s \in S$. Since $s \in S$, ______________. From the additive translation axiom for ordered fields, ______________. Hence, $c + \sup(S)$ is an upper bound for $c + S$.

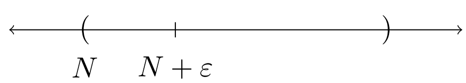
Next, we prove that $c+\sup(S)$ is the least upper bound for $c+S$. Let B be an upper bound for $c+S$. Then $B-c$ is an upper bound for S. To see this, let $s \in S$. Then ______________

Since $B - c$ is an upper bound for S and $\sup(S)$ is the *least upper bound*, ______________. It follows that $c + \sup(S) \leq B$, as desired. $\square$

PROBLEM 3. (We can approximate the infimum of a set arbitrarily closely with an element in the set.) Prove the following, imitating the proof of Proposition 4 in the reading for Friday, Week 5:

Let S be a subset of an ordered field F, and suppose that $N := \inf S$ exists. Given $\varepsilon \in F$ with $\varepsilon > 0$, there exists $s \in S$ such that $s - N < \varepsilon$.

Helpful picture. In trying to prove this result, the following picture (for the case $F = \mathbb{R}$ and S an interval) may help:



Note that (i) $s - N < \varepsilon$ is equivalent to $s < N + \varepsilon$ and (ii) since $N = \inf(S)$ and $s \in S$ it follows that s is in the interval $[N, N + \varepsilon)$.

PROBLEM 4. Prove that $\mathbb{C}$ satisfies the distributivity axiom for a field. (Work directly from our definition of $\mathbb{C}$ as $\mathbb{R}^2$ with a certain addition and multiplication, i.e., use the (a, b) notation as opposed the $a + bi$ notation.)

PROBLEM 5. Compute the following and express in the form $a + bi$ with $a, b \in \mathbb{R}$:

(a) $(2 - 7i)(1 + 2i) + (4 + i)$

(b) $\dfrac{1}{2 - i}$

(c) $\dfrac{1 + 3i}{3 + i}$

(d) $\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2} i\right)^3$