

PROBLEM 1. Let F be a field, and let $x, y \in F$. Prove that

$$(-x)y = -(xy).$$

Your proof should not use -1 . Instead, use the *definition of an additive inverse*. Justify all steps in your proof. (You may use that $a \cdot 0 = 0$ for all $a \in F$ since we proved that earlier.) Here is a template:

We need to show that is the additive inverse of . We check this as follows:

PROBLEM 2. Let F be a field, and let $x, y, z \in F$. Use the field axioms to show that if $x \neq 0$, then

$$xy = xz \implies y = z.$$

Justify each step of your argument.

PROBLEM 3. If x is an element of an ordered field, we define $x^1 := x$, and for each $n \geq 1$, we define $x^{n+1} := x \cdot x^n$.

Let F be an ordered field, and let $x \in F$ satisfy $x > 1$. Prove that $x^n > 1$ for all integers $n \geq 1$, being careful to cite relevant order axioms. (In our notes, we showed that $1 > 0$ in any ordered field. Since $x > 1$ and $1 > 0$, it follows by transitivity that $x > 0$. The fact that $x > 0$ will be relevant at some point in your proof. Also, since we have a recursive definition of x^n , a proof by induction would be natural.)