PROBLEM 1. Given a set A, the *power set* of A is the set $\mathcal{P}(A)$ of all subsets of A:

$$\mathcal{P}(A) = \{B\colon B \subseteq A\}.$$

Another notation for the power set is 2^A. If A has n elements, it turns out that $\mathcal{P}(A)$ has 2^n elements (to make a subset, go to each element of A and make one of two choices: the element is in the subset, or it is not).

(a) List the elements of $\mathcal{P}(A)$ in the case where $A = \{x, y, z\}$.
(b) For a *general* set A (not necessarily the set A in part (a)) is the subset relation $\subseteq$ on $\mathcal{P}(A)$ reflexive? Is it symmetric? Is it transitive? For each of these properties, either give a proof that it holds or provide a (simple, concrete) counterexample.
(c) Is the subset relation $\subseteq$ on $\mathcal{P}(A)$ an equivalence relation?

PROBLEM 2. Let $X = \{x, y, z\}$, and consider the relation

$$R = \{(x, y), (y, y), (y, z)\} \subset X \times X$$

(a) List the elements of the smallest possible equivalence relation on X that contains R, explaining why the elements you add to R are required.
(b) How many equivalence classes does the resulting equivalence relation have?

The next three problems all refer to the same relation, which we now describe. Consider the set $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ of ordered pairs of integers with non-zero second component. Define an equivalence relation $\sim$ on $\mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$ by $(a, b) \sim (a', b')$ if $ab' = ba'$. For instance $(2, 6) \sim (1, 3)$ since $2 \cdot 3 = 6 = 6 \cdot 1$. Problems 3, 4, and 5, below, all refer to this equivalence relation.

PROBLEM 3. Prove that $\sim$ is an equivalence relation. (Follow the template, and make sure in your proof that you do not potentially divide by 0.)

PROBLEM 4. For each of the following equivalence classes, give a general description of all of the elements in the equivalence class and then give five concrete examples of elements in the equivalence class: $[(0, 2)]$, $[(3, 3)]$, and $[(3, 5)]$.

PROBLEM 5. The rational numbers are the set $\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z} \text{ and } b \neq 0\right\}$, i.e., of ordinary fractions.

(a) Describe a one-to-one correspondence (a bijection) between the rational numbers and the set of equivalence classes for $\sim$. (The equivalence classes are all infinite. Hence, each fraction is going to correspond to an infinite list of pairs of integers under this correspondence.)
(b) Find integers a and b such that the fractions $6/15$ and $2/5$ both correspond to the equivalence class $[(a, b)]$ under your correspondence. (No explantion necessary.)

PROBLEM 6. Show that the function

$$
\begin{aligned}
f\colon \mathbb{R} &\mapsto \mathbb{R} \\
x &\mapsto 2x + 5
\end{aligned}
$$

is a bijection. (Do *not* do this by providing an inverse function. Instead, use our template: first prove it is injective, then prove it is surjective).