Math 112 Homework, Friday Week 2

Make sure to review the *Homework* portion of our Course Information sheet before writing up your solutions! For instance: **you will only receive full credit if you provide full explanations**. Also, your **solutions should consist *solely* of complete sentences**. Simply providing the correct numerical solution does not suffice. See the *Mathematical Writing* handout.

PROBLEM 1. Compute $\prod_{k=2}^{4}\left(1-\frac{1}{k^2}\right)$.

PROBLEM 2. Use induction to prove that

$$\prod_{k=2}^{n}\left(1-\frac{1}{k^2}\right)=\frac{n+1}{2n}.$$

for all $n \geq 2$.

PROBLEM 3. Let $A=\{1,3,5\}$, $B=\{4,5,6\}$, and $C=\{2,4,6,8\}$. Find the following sets:

(a) $(A \setminus B) \cup (B \setminus A)$.
(b) $C \setminus (B \setminus A)$.
(c) $C \cup (B \setminus A)$.
(d) $(C \cap A) \cup (C \cap B)$.
(e) $\{A\} \cap \{B\}$.

In this problem, your solution to (a) can take form "$(A \setminus B) \cup (B \setminus A) = \{$your answer here$\}$.", for example, and similarly for the other parts.

PROBLEM 4. Let X and Y be sets. Following the template given in class (see the video lecture or the end of our compiled lecture notes), prove that

$$(X \setminus Y) \cup (Y \setminus X) = (X \cup Y) \setminus (X \cap Y).$$

(Hint: you might want to split both parts of the proof into cases.)

PROBLEM 5. Let A, B, C, D be sets. Either prove the following or give an explicit counterexample showing that equality does not hold:

$$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D).$$