

Math 112 Homework for Friday, Week 1

1. Show by induction that for all  $n \geq 2$ ,

$$\prod_{k=2}^n \left(1 - \frac{1}{k^2}\right) = \frac{n+1}{2n}.$$

2. Show by induction that for all  $n \geq 1$ ,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

3. Let  $n$  lines be drawn in the plane in *general position*: no two are parallel and no three meet in a single point.

- (a) Let  $R(n)$  be the number of regions created by the  $n$  lines. Conjecture a formula for  $R(n)$  for  $n \geq 0$ . Note:  $R(0) = 1$  and  $R(1) = 2$ . (Hint: the formula is quadratic in  $n$ .)
- (b) Prove your formula using induction. (Imagine drawing the  $(n+1)$ -st line. How many regions does it add? You may use this fact without proof.)