Math 112 Group problems, Wednesday Week 13

PROBLEM 1. Let $a, b, c \in \mathbb{R}$ and

$$f(x) = x^3 + ax^2 + bx + c.$$

How does the intermediate value theorem guarantee the existence of $\alpha \in \mathbb{R}$ such that $f(\alpha) = 0$? A rigorous proof is not required. Generalize this result.

PROBLEM 2. Let $f(x) = x^3 - 3x + 1$. Use the intermediate value theorem to prove that the equation $f(x) = 0$ has at least three solutions in $\mathbb{R}$

PROBLEM 3. Slice the earth with a plane to get a circle. Use the intermediate value theorem to prove there are opposite points on this circle having the same temperature. (Describe the function to which you are applying the IVT and the assumptions that you are making in order for the hypotheses of the theorem to be satisfied.)

PROBLEM 4. Suppose you hike a loop trial in the Columbia River Gorge. You stop for lunch someplace along the way, neither at the highest nor the lowest point. Under what conditions can you guarantee that there is some other point along the trail with the same elevation? (Again, carefully describe the function to which you are applying the IVT, and check the hypotheses that must be satisfied.)

PROBLEM 5. Let $f\colon [a, b] \to \mathbb{R}$ be a continuous function, and suppose that $f(a) < 0$ and $f(b) > 0$. Describe an algorithm based on the intermediate value theorem that estimates a value $c \in (a, b)$ such that $f(c) = 0$. How quickly does this algorithm converge on a solution?