For convenience:

$\lim_{x\to a} f(x) = L$ if for all $\varepsilon > 0$ there exists $\delta > 0$ such that

$$0 < |x - a| < \delta \quad \Rightarrow \quad |f(x) - L| < \varepsilon.$$

PROBLEM 1. Find $\lim\limits_{x\to 9} x^2$, and provide an ε-δ proof.

PROBLEM 2. Find $\lim\limits_{x\to 3} \dfrac{1}{2+x}$, and provide an ε-δ proof.

PROBLEM 3. Find $\lim\limits_{x\to 1} \left(x^2 + 3x + 2\right)$, and provide an ε-δ proof.

PROBLEM 4. Define

$$f\colon \mathbb{R} \to \mathbb{R}$$
$$x \mapsto \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational.} \end{cases}$$

Does $\lim_{x\to 0} f(x)$ exist? If so, then provide an ε-δ proof. If not, then provide an ε that can't be beat by any δ.