PROBLEM 1. Apply the ratio test to each of the following series, and state what conclusion may be drawn:

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{n!}{5^n} \qquad \text{(b)} \quad \sum_{n=1}^{\infty} \frac{n^2}{(2n)!} \qquad \text{(c)} \quad \sum_{n=1}^{\infty} \frac{1}{2n^2} \qquad \text{(d)} \quad \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

For part (d), you may use the fact that $\lim_{n\to\infty}(1 + 1/n)^n = e$.

PROBLEM 2. Apply the integral test to each of the following series, and state what conclusion may be drawn:

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \qquad \text{(b)} \quad \sum_{n=1}^{\infty} \frac{1}{n^{4/3}} \qquad \text{(c)} \quad \sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}}$$

PROBLEM 3. As a consequence of our limit theorems, we know that if $\sum_n a_n$ and $\sum_n b_n$ converge, then so do $\sum_n (a_n + b_n)$ and $\sum_n ca_n$ for all constants c. It turns out that it is not necessarily true that $\sum_n a_n b_n$ converges. As a special case (where $a_n = b_n$), find a series $\sum_n a_n$ such that $\sum_n a_n = 0$, and yet $\sum_n a_n^2$ diverges to ∞.