PROBLEM 1. Let $f\colon \mathbb{C} \to \mathbb{C}$ be given by $f(z) = 3z^2 + 2$. Compute $f'(3i)$ directly from the definition of the derivative.

PROBLEM 2. Let $A, B, C \subseteq F$ where $F = \mathbb{R}$ or $\mathbb{C}$, and suppose that $f\colon A \to B$ and $g\colon B \to C$ are continuous functions. Show that $g \circ f$ is continuous by filling in the blanks below.

Proof. Let $a \in A$, and let $\varepsilon > 0$. Since g is continuous at $f(a)$, there exists $\delta > 0$ such that

(1) $\quad |x - f(a)| < \delta \quad \Rightarrow \quad$ [________].

Fix this δ. Since f is continuous at a, there exists $\eta > 0$ such that

(2) $\quad |x - a| < \eta \quad \Rightarrow \quad$ [________]

Combining (1) and (2), we see that

$|x - a| < \eta \quad \Rightarrow \quad$ [________]

Thus, $g \circ f$ is continuous at a. □

PROBLEM 3.

(a) Let $z, w \in \mathbb{C}$. What do the triangle inequality and the reverse triangle inequality say about $|z + w|$? What about $|z - w|$?

(b) Prove that the function $f\colon \mathbb{C} \to \mathbb{C}$ defined by $f(x) = |x|$ is continuous.