Math 112 Group problems, Friday Week 10

PROBLEM 1. Use the limit comparison test to determine whether the following series converge. You may use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2 - 6n + 3}{4n^6 + n^3 + 7}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 2}}$

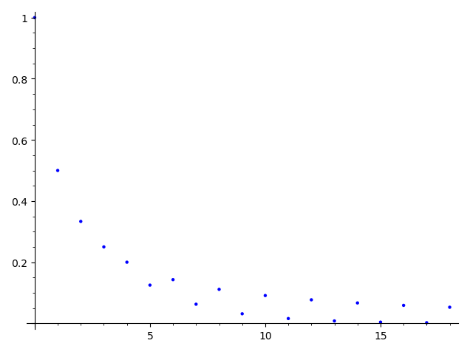
PROBLEM 2. Are the following series absolutely convergent, conditionally convergent, or divergent?

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{3n+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+4}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{5^{n+1}}$.

PROBLEM 3. What does the alternating series test say about the following series?

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{2^2} + \frac{1}{5} - \frac{1}{2^3} + \frac{1}{7} - \frac{1}{2^4} + \cdots$$

Here is a plot of the first few partial sums, $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{2^2}, \frac{1}{5}, \ldots$:



PROBLEM 4. Consider the series from the previous problem:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{2^2} + \frac{1}{5} - \frac{1}{2^3} + \frac{1}{7} - \frac{1}{2^4} + \cdots. \tag{1}$$

Here is a typical partial sum:

$$s_{2k+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{2^2} + \cdots - \frac{1}{2^k} + \frac{1}{2k+1}$$

$$= 1 + \frac{1}{3} + \cdots + \frac{1}{2k+1} - \left(\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^k}\right).$$

(a) Prove that $\sum_{k=0}^{\infty} \frac{1}{2k+1}$ diverges to infinity.

(b) Find a lower bound for s_{2k+1} that allows you to show that the series (1) diverges.

(c) Why doesn't this example violate the alternating series test?