

*Math 112 Group problems, Wednesday Week 7*

PROBLEM 1. Let  $a_n = (-1)^n$ , and let  $a = 0$ . Is the following statement true or false? Provide a proof or explicit counterexample.

For all  $N \in \mathbb{R}$  there is an  $\varepsilon > 0$ , such that if  $n > N$ , then  $|a - a_n| < \varepsilon$ .

What is the relevance of the above statement to the question of the convergence or divergence of  $\{a_n\}$ ?

PROBLEM 2. Let  $a_n = 1/n$  for  $n \geq 1$ , and let  $a = 0$ . Is the following statement true or false? Provide a proof or explicit counterexample.

For all  $\varepsilon > 0$  and  $N \in \mathbb{R}$ , if  $n > N$ , then  $|a - a_n| < \varepsilon$ .

What is the relevance of the above statement to the question of the convergence or divergence of  $\{a_n\}$ ?

PROBLEM 3. Find the limit of  $\lim_{n \rightarrow \infty} \frac{3n^3 + 2n}{6n^3 + 4n + 7}$  and provide an  $\varepsilon$ - $N$  proof.

PROBLEM 4. (Challenge, if there is extra time.) Prove that  $\lim_{n \rightarrow \infty} \frac{1}{n} \neq 1$ . (Hint: you need to find an explicit  $\varepsilon > 0$  that can't be beat by any  $N \in \mathbb{R}$ .)