Math 112 Group problems, Friday Week 7

PROBLEM 1. Give an ε-N proof that

$$\lim_{n\to\infty} \frac{\cos(n) + \sqrt{2}i\sin(n)}{n} = 0.$$

(Hint: the triangle inequality is your friend.)

PROBLEM 2. Give an ε-N proof that

$$\lim_{n\to\infty} \frac{n}{4n^3 + 2n^2 + 5n + 1} = 0.$$

PROBLEM 3. Give an ε-N proof that

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0.$$

PROBLEM 4. Does the sequence $\left\{\sqrt{n+1} - \sqrt{n}\right\}$ converge? Proof?

PROBLEM 5. (Challenge, if you have extra time.)

Does $\{\frac{n!}{n^n}\}$ converge? (Hint: write $n!/n^n$ as a product of n distinct factors, and try to bound it above by a nice function of n.)