

*Math 112 Group problems, Monday Week 6*

PROBLEM 1. (Square roots of  $-1$ .) For  $n \in \{2, 3, 4, 5, 6, 10\}$ , find all  $x \in \mathbb{Z}/n\mathbb{Z}$  such that  $x^2 = -1$ .

PROBLEM 2. Prove that  $\mathbb{C}$  satisfies the additive associativity axiom. (Use the definition of  $\mathbb{C}$ , taking one step at a time, justifying each step. You will need to use the definition of addition for  $\mathbb{C}$  and associativity of addition for  $\mathbb{R}$ .)

PROBLEM 3. Consider the set  $\mathbb{R}^2$  with addition and multiplication defined by

$$(a, b) + (c, d) = (a + c, b + d) \quad \text{and} \quad (a, b) \cdot (c, d) = (ac, bd),$$

respectively. Indicate which field axioms fail, giving a concrete counter-example in each case. What are the additive and multiplicative identities? (To save time, you may assume the fact that associativity of addition and multiplication hold.)

PROBLEM 4. Compute the following, expressing your result in the form  $a + bi$  for  $a, b \in \mathbb{R}$ .

(a)  $(3 + 2i)(-2 + 3i) + (1 + 4i)$

(b)  $(2 + 3i)^{-1}$

(c)  $\frac{1+4i}{2+i}$ .