Math 112 Group problems, Monday Week 5

Problem 1. Let F be an ordered field, and let $w, x, y, z \in F$. Use the order axioms to prove that if $w < x$ and $y < z$, then $w+y < x+z$. In other words, we can "add inequalities".

Problem 2. Let F be an ordered field, and let $x \in F$ with $x > 0$. Since F is a field, x has a multiplicative inverse, $1/x$. Prove that $1/x > 0$. [Hint: break the possibilities for $1/x$ into cases using trichotomy, and rule out two of those cases.]

Problem 3. Can the field $\mathbb{Z}/5\mathbb{Z}$ be ordered? In other words, does there exist a relation on $\mathbb{Z}/5\mathbb{Z}$ satisfying the order axioms? [Hint: from the lecture notes, we know that for any nonzero element x of an ordered field, we have $x^2 > 0$. In particular, this means that $1 > 0$ since $1 = 1^2$. Start with $1 > 0$.]