Problem 1.

(a) If $F$ is a field and $x$ is a nonzero element of $F$. What is the meaning of $\frac{1}{x}$ (also denoted $x^{-1}$)?

(b) What is $\frac{1}{3}$ in the field $\mathbb{Z}/7\mathbb{Z}$? (Denote the equivalence classes for $\mathbb{Z}/7\mathbb{Z}$ be $\{0, 1, 2, 3, 4, 5, 6\}$, for convenience.)

(c) Show that 2 does not have a multiplicative inverse in $\mathbb{Z}/8\mathbb{Z}$.

Problem 2. Let $F$ be a field. In the lecture notes, we proved that for any $x \in F$, we have $x \cdot 0 = 0$. Using that proof as a model, prove that if $x, y, z \in F$, then

$$z + x = z + y \implies x = y.$$

Your proof should proceed by using one axiom per step. (You will need A4, A2, and the definition of 0 (A3).) The above result is called the *cancellation law for addition* in a field.

Problem 3. Let $F$ be a field, and let $x \in F$

(a) What is the meaning of $-x$?

(b) What is $-3$ in the field $\mathbb{Z}/7\mathbb{Z}$? (Again, denote the equivalence classes for $\mathbb{Z}/7\mathbb{Z}$ be $\{0, 1, 2, 3, 4, 5, 6\}$.)

(c) Prove that $-1 \cdot x = -x$. (You will need to focus on the definitions of $-1$ and $-x$. Since $F$ is a field, it has a multiplicative identity 1, and that multiplicative identity must, like all element of $F$, have an additive inverse, $-1$. By definition, $-1$ is the element of $F$ which when added to 1 yields the additive identity, 0. To test if a field element is $-x$, you add it to $x$ and see if you get 0. You will also probably use the fact that $0 \cdot x = 0$, which we proved in the lecture notes.)