PROBLEM 1. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$.

(a) Give an example of two subsets X and Y of A and a function $f\colon A \to B$ such that $f(X \cap Y) \neq f(X) \cap f(Y)$.

(b) Is it possible to find subsets X and Y of A such that $f(X \cap Y) \not\subseteq f(X) \cap f(Y)$?

PROBLEM 2. For each of the following functions, state why there is no inverse, or describe the inverse function.

(a)

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto |x|. \end{aligned}$$

(b)

$$\begin{aligned} g\colon \mathbb{R}_{\geq 0} &\to \mathbb{R} \\ x &\mapsto |x|. \end{aligned}$$

(c)

$$\begin{aligned} h\colon \mathbb{R}_{\geq 0} &\to \mathbb{R}_{\geq 0} \\ x &\mapsto |x|. \end{aligned}$$

PROBLEM 3. Show that the function

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto 3x + 1 \end{aligned}$$

is a bijection by providing its inverse function. (Demonstrate that the function you produce is actually the inverse of f. You need to check both possible compositions are the identity.) [This is the second method of proving that a function is bijective. The first, which more closely follows the definition of bijectivity is to prove that the function is both injective and surjective.]

PROBLEM 4. Consider the functions $f(x) = x + 1$ and $g(x) = 3x$, both with domain and codomain $\mathbb{R}$. Compute the following: (i) $g \circ f$, (ii) $(g \circ f)^{-1}$, (iii) f^{-1}, (iv) g^{-1}, and (v) $f^{-1} \circ g^{-1}$. Verify that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.