Math 112 Group problems, Friday Week 3

PROBLEM 1. Fill in the following addition and multiplication tables (using standard representatives for equivalence classes for convenience, e.g, 3 instead of [3]).

$\mathbb{Z}/5\mathbb{Z}$

+	0	1	2	3	4
0					
1					
2					
3					
4					

·	0	1	2	3	4
0					
1					
2					
3					
4					

$\mathbb{Z}/6\mathbb{Z}$

+	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

·	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

PROBLEM 2. Why are all of the tables in the previous problem symmetric about the diagonal from top-left to bottom-right? Do you see any other patterns?

PROBLEM 3. Let $a, b \in \mathbb{Z}$. When is $a = b \bmod 2$? When is $a = b \bmod 1$? When is $a = b \bmod 0$? List the equivalence classes in each case, i.e., the elements of $\mathbb{Z}/n\mathbb{Z}$, for $n = 2, 1, 0$.

PROBLEM 4. Use modular arithmetic to find the last two digits of the following two numbers:

$$101^{(10^{1000}+2021)} \quad \text{and} \quad 99^{(10^{1000}+2021)}.$$

PROBLEM 5 (Challenge). Let $a_1 = 3$, and for $n > 0$, define $a_n = 3^{a_{n-1}}$. Thus, $a_2 = 3^3 = 27$, and $a_3 = 3^{3^3} = 3^{27}$. What is the last digit of a_{100}? (Hint: start by considering the last digits of 3, 3^2, 3^3, 3^4, etc., until you see a pattern. You may start to think that the number 4 is significant.)