PROBLEM 1. Let A be the set of all lines in the plane. Is the relation "is parallel to" on A an equivalence relation? If not, which properties prevent if from being so. (Take "parallel" to mean "same slope" rather than "non-intersecting". How does this affect your answer?)

PROBLEM 2. Let A be the set of all lines in the plane. Is the relation "is perpendicular to" on A an equivalence relation? If not, which properties prevent if from being so.

PROBLEM 3. For $a, b \in \mathbb{Z}$, say $a \sim b$ if $a - b = 2k$ for some $k \in \mathbb{Z}$. In other words, $a \sim b$ if $a - b$ is an even integer. Prove that $\sim$ is an equivalence relation on $\mathbb{Z}$ following the template below:

Theorem. Define a relation $\sim$ on a set A by blah, blah, blah. Then $\sim$ is an equivalence relation.

Proof. Let $a, b, c \in A$.

Reflexivity. We have $a \sim a$ since blah, blah, blah. Therefore, $\sim$ is reflexive.

Symmetry. Suppose that $a \sim b$. Then, blah, blah, blah. It follows that $b \sim a$. Therefore $\sim$ is symmetric.

Transitivity. Suppose that $a \sim b$ and $b \sim c$. Then blah, blah, blah. It follows that $a \sim c$. Therefore, $\sim$ is transitive.

Since $\sim$ is reflexive, symmetric, and transitive, it follows that $\sim$ is an equivalence relation. $\square$