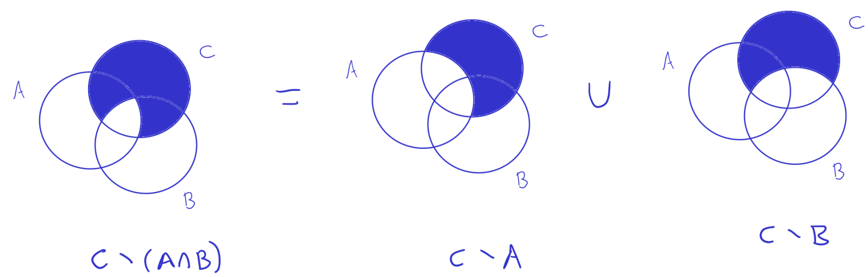
Problem 1. **Proposition.** Let A, B, C be sets. Then

$$C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B).$$

(a) Draw a Venn diagram that shows the Proposition is reasonable.
(b) Prove the Proposition.

Solution:

(a)



(b) Let $x \in C \setminus (A \cap B)$. Then $x \in C$ and $x \notin A \cap B$. Since $x \notin A \cap B$, then x is not in both A and B. Thus, we have two cases to consider. First, say $x \notin A$. Then we have that $x \in C$ and $x \notin A$. Thus, $x \in C \setminus A$, and, hence, $x \in (C \setminus A) \cup (C \setminus B)$, as desired. Next, if $x \notin B$, then $x \in C \setminus B$, and it again follows that $x \in (C \setminus A) \cup (C \setminus B)$.

Conversely, now assume that $x \in (C \setminus A) \cup (C \setminus B)$. Therefore, $x \in C \setminus A$ or $x \in C \setminus B$. Again, we have two cases. First, suppose $x \in C \setminus A$. Then $x \in C$ and $x \notin A$. It follows that $x \in C$ and $x \notin A \cap B$. Therefore, $x \in C \setminus (A \cap B)$. The second case follows similarly.

Problem 2. Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Write all of the elements of $A \times B$.

Solution: The elements of $A \times B$ are:

$$(1, a), (1, b), (1, c), (2, a), (2, b), (2, c).$$

Problem 3. Let A, B, C be sets. Show that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

Solution:

Proof. Let $(x, y) \in A \times (B \cap C)$. Then $x \in A$ and $y \in B \cap C$. Since $y \in B \cap C$, it follows that $y \in B$ and $y \in C$. Since $x \in A$ and $y \in B$, it follows that $(x, y) \in A \times B$. Since $x \in A$ and $y \in C$, it follows that $(x, y) \in A \times C$. Therefore $x \in (A \times B) \cap (A \times C)$.

Conversely, suppose that $(x, y) \in\in (A\times B)\cap(A\times C)$. Then $(x, y) \in A\times B$ and $(x, y) \in A\times C$. We conclude that $x \in A$ and that y is in both B and C, i.e., $y \in B \cap C$. Hence, $(x, y) \in A \times (B \cap C)$. $\square$