PROBLEM 1. Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$ Find the following:

(a) $A \cup B \cup C$
(b) $A \cap B \cap C$
(c) $A \setminus B$
(d) $B \setminus A$
(e) $(A \cup B) \cap C$
(f) $(A \cap C) \cup (B \cap C)$
(g) $(A \cap B) \cup C$
(h) $(A \cup B) \cap (A \cup C)$.

SOLUTION:

(a) $A \cup B \cup C = \{1, 2, 3, 4, 5\}$
(b) $A \cap B \cap C = \{3\}$
(c) $A \setminus B = \{1\}$
(d) $B \setminus A = \{4\}$
(e) $(A \cup B) \cap C = \{1, 2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$
(f) $(A \cap C) \cup (B \cap C) = \{3\} \cup \{3, 4\} = \{3, 4\}$
(g) $(A \cap B) \cup C = \{2, 3\} \cup \{3, 4, 5\} = \{2, 3, 4, 5\}$
(h) $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4\} \cap \{1, 2, 3, 4, 5\} = \{1, 2, 3, 4\}$.

PROBLEM 2. Suppose that A, B, C are sets with $A \subseteq B \subseteq C$. Prove or disprove:

$$C \setminus B \subseteq C \setminus A.$$

SOLUTION: Let $x \in C \setminus B$. Then $x \in C$ and $x \notin B$. Since $A \subseteq B$ and $x \notin B$, it follows that $x \notin A$. Thus, $x \in C$ and $x \notin A$. It follows that $x \in C \setminus A$. Therefore. $C \setminus B \subseteq C \setminus A$.

PROBLEM 3. Let $A = \{1, \{3, 6, 9\}, \{\emptyset\}\}$.

(a) What are the elements of A?
(b) Is $6 \in A$?
(c) Is $\{1\} \subseteq A$?
(d) Is $\emptyset \subseteq A$?
(e) Is $\emptyset \in A$?

SOLUTION:

(a) The set A has three elements: 1, $\{3, 6, 9\}$, and $\{\emptyset\}$.
(b) No, but 6 is an element of the element $\{3, 6, 9\}$ of A.
(c) Yes, every element of $\{1\}$ is an element of A.

(d) Yes, the empty set is a subset of every set. (The statement that every element of the empty set is a subset of A is *vacuously* true. For this statement to be false, the empty set would need to contain an element.)

(e) No: $\emptyset$ is not one of the three elements of A. On the other hand, it is true that $\{\emptyset\} \in A$.

PROBLEM 4. Describe the following intersection of open intervals of $\mathbb{R}$

$$\bigcap_{n=1}^{\infty}(-1/n, 1/n) = (-1,1) \cap (-1/2, 1/2) \cap (-1/3, 1/3) \cap (-1/4, 1/4) \cap \cdots$$

as simply as possible. What are its elements? Do the same for

$$\bigcup_{n=1}^{\infty}(-1/n, 1/n) = (-1,1) \cup (-1/2, 1/2) \cup (-1/3, 1/3) \cup (-1/4, 1/4) \cup \cdots .$$

SOLUTION: The only element that is $(-1/n, 1/n)$ for all integers $n \geq 1$ is 0. Therefore,

$$(-1,1) \cap (-1/2, 1/2) \cap (-1/3, 1/3) \cap (-1/4, 1/4) \cap \cdots = \{0\} .$$

We have

$$(-1,1) \cup (-1/2, 1/2) \cup (-1/3, 1/3) \cup (-1/4, 1/4) \cup \cdots = (-1,1).$$