Problems

1. For each of the following languages, give a regular expression that represents the language. In all cases $\Sigma = \{0,1\}$.
   (a) $L = \{w \mid |w| \leq 5\}$
   (b) $L = \{w \mid w$ does not contain the substring 001$\}$

2. Show that the class of regular languages is close under intersection. That is, if $A$ and $B$ are both regular languages, then so is $A \cap B$.

3. For each of the following languages, prove either that it is regular or that it is not regular. In all cases $\Sigma = \{0,1\}$.
   (a) $L = \{w \mid w$ contains an equal number of 0s and 1s$\}$
   (b) $L = \{1^k y \mid y \in \Sigma^*, k \geq 1$, and $y$ contains at least $k$ 1s$\}$
   (c) $L = \{1^k y \mid y \in \Sigma^*, k \geq 1$, and $y$ contains at most $k$ 1s$\}$

4. Consider the language $L = \{0^i1^j2^k \mid i, j, k \geq 0$ and if $i = 1$ then $j = k\}$.
   (a) Show that $L$ is not regular.
   (b) Show that $L$ does not look irregular as far as the pumping lemma goes. That is, give a pumping length $p$ and show that $L$ satisfies the conditions of the pumping lemma.
   (c) Explain why the two things you’ve shown above do not contradict.

Bonus problems

1. Let $A/B = \{w \mid wx \in A$ for some $x \in B\}$. Show that if $A$ and $B$ are regular, then $A/B$ is regular.

2. Our goal in this problem is to show that the representation of objects can affect whether or not a given set can be recognized by a machine. Consider a set $A$ of natural numbers. Let $B_k(A)$ be the set of strings that represent numbers from $A$ in base $k$ (with no leading zeros). For example, if $A = \{3, 5\}$ then $B_2(A) = \{11, 101\}$ and $B_3(A) = \{10, 12\}$. We can think of $B_k(A)$ as a language with a $k$-symbol alphabet. Give a set $A$ for which $B_2(A)$ is regular but $B_3(A)$ is not (and prove it).