01. Let \( f \) be the complex valued function defined as follows:
\[
f(z) = \tan\left(\frac{1}{2i} \log\left(\frac{1+iz}{1-iz}\right)\right)
\]
Describe the “natural” domain \( \Omega \) for \( f \). Show that:
\[
f(z) = z \quad (z \in \Omega)
\]

02. Evaluate the integral:
\[
\int_0^{2\pi} \frac{1}{a + b \sin \theta} d\theta
\]
where \( a \in \mathbb{R}, b \in \mathbb{R}, \) and \( 0 < |b| < a \).

03. Evaluate the contour integrals:
\[
\int_{\Gamma} \frac{z \exp(z)}{z + 2i} dz, \quad \int_{\Delta} \frac{z \exp(z)}{z + 2i} dz
\]
where:
\[
\Gamma(t) = \exp(it), \quad \Delta(t) = 3 \exp(it), \quad 0 \leq t \leq 2\pi
\]

04. Let \( f \) be the complex valued function defined as follows:
\[
f(z) = \frac{1}{(z - 2)z(z + 1)}
\]
where \( 1 < |z| < 2 \). Find the Laurent Expansion for \( f \) in the annulus on which it is defined.

05. Let \( f \) be a complex valued function defined and analytic on the entire complex plane \( \mathbb{C} \). For each positive real number \( r \), let:
\[
M(r) = \max_{|z|=r} |f(z)|
\]
Show that, for any positive real numbers \( r' \) and \( r'' \):
\[
r' < r'' \implies M(r') < M(r'')
\]
06• Determine the number of complex numbers \( \zeta \) for which \( 1 < |\zeta| < 2 \) and:
\[
\zeta^4 - 6\zeta + 3 = 0
\]

07• Let \( \Omega \) be the region in \( \mathbb{C} \) defined as follows:
\[
z \in \Omega \iff [(0 < x) \text{ and } (x \leq 1 \implies y \neq 0)] \quad (z = x + iy)
\]
Let \( f \) be the complex valued function defined on \( \Omega \) as follows:
\[
f(z) = i\sqrt{z^2 - 1} \quad (z \in \Omega)
\]
Confirm that \( f \) is analytic. Describe the range of \( f \). Let \( u \) and \( v \) be the real and imaginary parts of \( f \):
\[
f(z) = w = u(x, y) + iv(x, y)
\]
Sketch the level sets for \( u \) and \( v \):
\[
u(x, y) = a, \quad v(x, y) = b
\]

08• Let \( \Omega \) be a region in \( \mathbb{C} \) of the following form:
\[
\Omega = \Omega^+ \cup J \cup \Omega^-
\]
where \( \Omega^+ \) is a region in \( \mathbb{C} \) such that:
\[
z \in \Omega^+ \implies 0 < y \quad (\text{where } z = x + iy)
\]
where \( \Omega^- \) is the region in \( \mathbb{C} \) conjugate to \( \Omega^+ \):
\[
z \in \Omega^- \iff \bar{z} \in \Omega^+
\]
and where \( J \) be an open interval in \( \mathbb{R} \). (Review the definition of a region.)
Let \( f \) be a complex valued function defined and analytic on \( \Omega^+ \) such that, for each (real) number \( u \) in \( J \):
\[
\lim_{z \to u} f(z) = 0
\]
Show that, for each (complex) number \( z \) in \( \Omega^+ \), \( f(z) = 0 \). To that end, introduce the complex valued function \( \phi \), defined on \( \Omega \) as follows:
\[
z \in \Omega \implies \phi(z) = \begin{cases} f(z) & \text{if } z \in \Omega^+ \\ 0 & \text{if } z \in J \\ \frac{f(\bar{z})}{f(z)} & \text{if } z \in \Omega^-
\end{cases}
\]
Show that \( \phi \) is analytic. Finish the argument.
Find all solutions of the following equation:

\[ f''(z) + zf(z) = 0 \]

To that end, consider functions defined by power series.