1. For each of the following probability distributions, find the maximum likelihood estimates. Then compute the asymptotic standard error.

   (a) $X_i,\ i = 1 \ldots n \ \sim\ IID$ Unif$(0, \theta)$. Explain why the Uniform is not a 'regular family'(see course notes pages 74-75).

   (b) $X_i\ i = 1 \ldots n\ \sim\ IID$ Poisson(μ).

   (c) $X\ \sim$ Negative Binomial(r, p), where r is known. Compare to a Binomial(n, p), inparticular think about how the numbers of successes and trials affect the likelihood.

   (d) $X_i\ i = 1 \ldots n\ \sim\ IID$ Exponential(λ).

2. $X_i\ i = 1 \ldots n\ \sim\ IID$ Exponential(λ), but observations larger than T are censored. In other words, if $X_i > T$ we can't observe its value. Suppose that k of the $X's$ are observed, and $n - k$ are censored. The likelihood for λ may be written as follows:

$$L(\lambda) = \prod_{i=1}^{k} f(X_i \mid \lambda) \prod_{i=k+1}^{n} (1 - F(T \mid \lambda))$$

   where f is the density (likelihood!) and F is the CDF.

3. Let $X_1, X_2, \ldots X_k$ be the counts for n independent observations on a multinomial distribution with k categories. Note that

$$X_1 + X_2 + \ldots X_k = n.$$

   Find the MLE's for the probabilities of the k categories: $P_1, P_2, \ldots P_k$. (Note that $P_1 + P_2 + \ldots + P_k = 1$.