

Let  $G$  be a finite group

"Towards the equivariant Dyer-Lashof algebras"

"Equivariant" = "with/respecting a  $G$ -action"

5/30/15

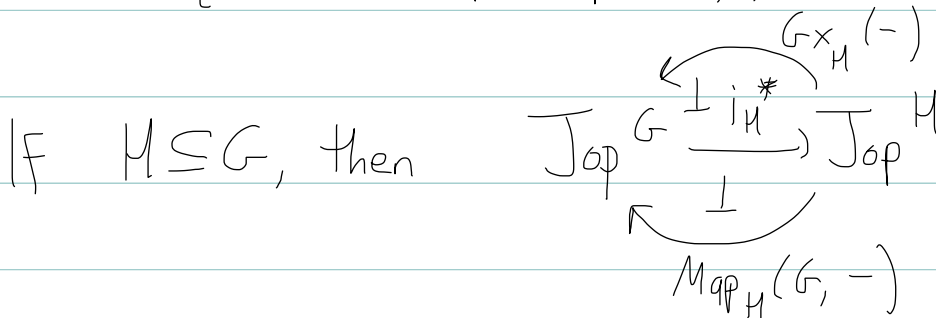
$$X \text{ a } G\text{-space} \iff G \rightarrow \text{Homeo}(X)$$

$\mathcal{J}$  objects: all  $G$ -spaces of  $G$  on  $X$   
 $\mathcal{J}_{op}$ : Maps: all maps w/ conjugation action

$$\text{Map}(X, Y) \ni G$$

$$f \mapsto gfg^{-1}$$

$\text{Top}^G = G\text{-spaces} \&$   
 equivariant maps  $\text{Map}(X, Y)^G$



In alg, left and right adjoint are isomorphic

Not true in spaces

$$H = \{e\} \subseteq G \quad G \times_{\{e\}} X \quad \text{This is disconnected}$$

What are  $[S^k, G \times_{\{e\}} X]^G = [S^k, (G \times_{\{e\}} X)^G]$

↑ trivial action

"∅"

$$[S^k, \text{Map}_{\xi e_3}(G, X)]^G \cong [L_{\xi e_3}^* S^k, X]^{\xi e_3} \cong \pi_k X$$

Stabilize by inverting all representation spheres

$$\text{In } \mathcal{S}_p^G = \begin{matrix} \text{stabilized} \\ G\text{-spaces,} \end{matrix} \quad \text{have} \quad G_+ \wedge_H X \xrightarrow{\sim} \text{Map}_H(G_+, X)$$

In  $\text{Jop}^G$  homotopy groups extend to functors

$$\begin{matrix} \text{finite} \\ G\text{-sets} \end{matrix} = (\text{Set}^G)^{\text{op}} \longrightarrow \text{Ab}$$

$$T \longrightarrow [T_+ \wedge S^k, X]^G =: \underline{\pi}_k(X)(T)$$

$$\text{Have functors} \quad \sum^\infty \text{Jop}^G \begin{matrix} \xrightarrow{\quad} \\ \xleftarrow{\quad} \end{matrix} \mathcal{S}_p^G : \Omega^\infty$$

$$\Omega^\infty \Sigma^\infty(X) = \varinjlim \Omega^{V_n} \Sigma^{V_n} X$$

$V_n$  is a seq of rep that eventually contains all finite dim'l reps

$\Omega^\infty \Sigma^\infty(X)$  has an action of an operad  $\mathcal{D}$

Standard  
etc

$\mathcal{D}(V_n)_K =$  Embeddings of  $\coprod_K D(V_n)$  into  $D(V_n)$

Dyer-Lashoff: operations in homology of an infinite loop space

$\mathcal{D}_K =$  Embeddings of  $\coprod_K D$  into  $D$  ↙ little disk in  $\oplus V_n$

has a  $G \times \Sigma_K$ -action

$$\Lambda \subseteq G \times \Sigma_K \quad \mathcal{D}_K^\wedge \cong \begin{cases} \emptyset & \Lambda \cap \Sigma_K \neq \{e\} \\ * & \Lambda \cap \Sigma_K = \{e\} \end{cases}$$

$\Rightarrow$  If  $\Lambda \cap \Sigma_K = \{e\}$  have a unique map  $G \times \Sigma_K / \Lambda \rightarrow \mathcal{D}_K$   
 $\parallel$   
 $(G \times \Sigma_K)_{\Lambda}^* *$

If  $Y = \Omega^\infty \Sigma^\infty(X)$ , then have  $\mathcal{D}_K \times_{\Sigma_K} Y^{\times K} \rightarrow Y$

$\Rightarrow G \times \Sigma_K / \Lambda \times_{\Sigma_K} Y^{\times K} \rightarrow Y$

$\Lambda \cap \Sigma_k = \{e\} \Rightarrow \exists H \subseteq G$  and  $f: H \rightarrow \Sigma_k$  s.t.

$$\Lambda = \Gamma_f = \{ (h, f(h)) \mid h \in H \}$$

Thus  $\Lambda / \text{conj} \iff H$  set structures on  $\{1, \dots, k\}$   $T_\Lambda$

$$(G \times \Sigma_k / \Lambda) \times_{\Sigma_k} Y^{\times k} \cong G \times_H \text{Map}(T, L_H^* Y) \text{ i.e. } D\text{-action}$$

gives maps  $G \times_H \text{Map}(T, L_H^* Y) \rightarrow Y$

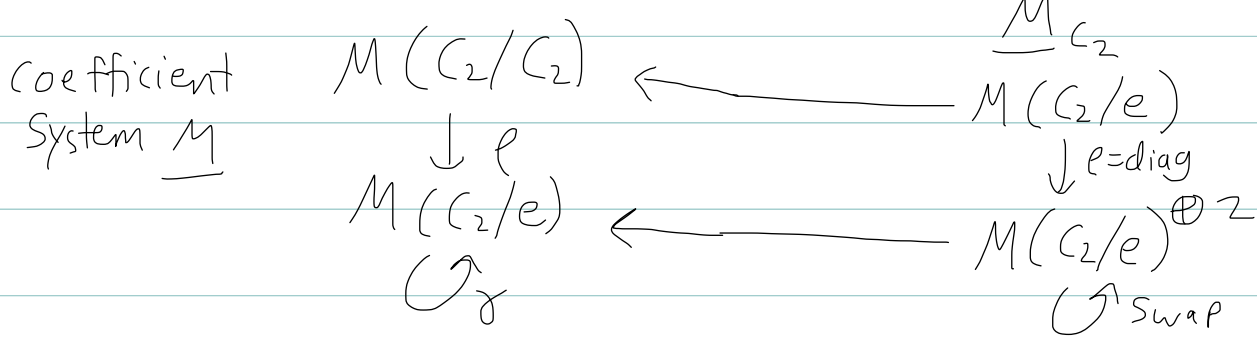
IF  $H=G$   $\text{Map}(T, Y) \rightarrow Y$  Apply  $\underline{\Pi}_k$

$\Lambda, m, k$   
 $G \times_H (H \times \Sigma_k / \Lambda)$   
 $\cong$   
 $(G \times \Sigma_k) / \Lambda$

$$(S \mapsto [(S \times T)_+ \wedge S^{k'}, Y]^G) \rightarrow \underline{\Pi}_{k'}(Y)$$

$$\cong \underline{\Pi}_{k'}(Y)_T$$

$G=C_2$  claim data is same data as Mackey functor



This recovers rest o trans =  $\sum_{\text{weil}}$

This is double coset formula

$$\text{If } T = * \amalg *$$

$$\text{Map}(* \amalg *, Y) \cong Y \times Y$$

$$\begin{array}{c} \downarrow \\ Y \end{array} \leftarrow \begin{array}{c} \text{m} \\ \text{mult} \end{array}$$

mult o swap = mult in homotopy

$\Rightarrow$  D algebra has mult m and maps  $M(G/H, Y) \rightarrow Y$   
"transfer"

• Replace  $Y$  by cat of  $\text{Top}_G$  gives model for  $G$  spectra  
(Guillou, Merling, Osorno)

Bredon homology  $Y \mapsto H_*(Y, \underline{M})$   
 $* \in RO(G)$

Dyer-Lashoff algebra:  $H_*(\mathcal{D}_k/\mathcal{L}_k; \underline{M})$  if  $\underline{M}$  is rational then have alg description

Q DL algebra is generated by  $m$  & all transfers

Need  $H_*(-; \underline{M})$  to be represented by a commutative ring Spectrum.

If  $M$  is not rat'l and  $|G|$  not invertible then get a copy of traditional Dyer-Lashoff alg for each? Now get odd Adem relations.

Motivic theory missing  $\Sigma^\infty$

Do Hu give way of tensoring with a space.

Q: (Tony Elmendorf) Missing classically that Homology of spaces in  $\mathcal{D}_k$  are homology of  $\Sigma_k$

A: Yes. Representations of  $\Sigma_k$  in Mackey functors have wrong notion of projective

Need resolution of Burnside Mackey functor

• every ratl Mackey functor is projective