Mathematics Writing Workshop

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(Topics or questions to be incorporated in the presentation?)

Yes, not all intelligence is verbal or quantitative.

It can be sensory, social, moral, esthetic, competitive, and so on.

But expressing thoughts in words is our currency in the liberal arts education environment.

And expressing thoughts in words *can* in fact have sensory, social, moral, esthetic, etc. aspects.

The act of writing is grand in scope.

Why try to write mathematics well?

One easily can avoid it.

On the face of it, quick and sloppy writing is easier:

- It saves time.
- It saves effort.
- Prettying things up is a waste of time once you understand.
- This is math (or science), not English class.
- And professors and graders don't always have the energy and the vigilance to bust you.

But *truly*, these arguments are short-sighted.

It is worth writing well for *one's own sake*, in many ways.

The goal is to become able to think clearly and gracefully.

Quality of thoughts and quality of their expression in words are inextricable.

Writing well is part of the work of learning to think well.

The often heard claim, "*I understand it, I just don't know how to explain it*" is equally bankrupt from students and from allegedly-brilliant expositors.

Admittedly, natural talent helps, but no matter who you are, it carries you only so far, and only then does the true engagement with the subject begin.

The secret to doing anything artfully and gracefully is **continual, effective practice**. There is no short cut.

Writing is part of study.

Inevitably, quick, sloppy writing will eventually render you unable to proceed any further into the subject, because you no longer can think about it clearly.

Thinking about mathematics is on opportunity to synthesize one's capacities with

- Geometry — here we can use our quick visual instincts.

- Algebra — here we can use our symbolic skills to grasp patterns that are wide-ranging and robust.

- Algorithms — here we can use our grasp of dynamic processes.

- Natural language — and here we can craft slogans to encapsulate central ideas, these concise encodings allowing us a detail-free large-scale view of the landscape.

How is mathematics usually written?

Terribly.

What is it like to read?

Often boring at best. Or even unintelligible, painful, demoralizing, enraging.

Why is this?

The author is sloppy.

The author is not writing for the reader.

The author is in a rush.

The author has lost touch with the reader's mindset.

The author does not make good choices of how to convey ideas.

How do these things happen? How to avoid them?

The author is sloppy.

There is simply no excuse for this. Ever. All that is great and good in writing requires an ongoing concern for technique.

Mathematical exposition is composed in an *extended* natural language, not an *alternative* to natural language. In the extended language, as in basic language, words and symbols have agreed-upon meanings. Do not re-invent meanings for symbols. Write in complete sentences. Use correct grammar. Choose your words well.

Once you are fluent with the rules and conventions, and only then, start to break them enlighteningly rather than nihilistically.

The author is not writing for the reader.

Decide who your reader is, and always keep your reader in mind.

The reader could be your near-future self trying to re-synthesize ideas quickly in preparation for an exam.

The author is in a rush.

This is hard to avoid. There is never enough time for anything.

Most mathematics texts are written between midnight and 2:00 a.m.

But still, starting work early, or even just looking over work early to let the ideas sit in your mind can help.

The author has lost touch with reader's mind-set.

To write well is to practice the humanizing skill of **empathy**. One can get better at this over time.

The author does not make good choices of how to convey ideas.

Much mathematical writing is done in **core dump** mode.

And above all –

It is boring.

The bulk of mathematical exposition proceeds in a *monotonous linearity*.

OF COURSE it is boring.

Who on earth would want to hear a story or watch a movie so presented?

Who on earth truly thinks linearly, at a uniform pace?

The Euclidean model of proceeding cumulatively from axioms is a crucial logical paradigm, but pedagogically its value lies more in its emphasis on *being as explicit as possible about one's assumptions* than on *linearity*.

How to do better?

Structurally:

- Introduction, overview.
- Foreshadowing
- Variable pacing, emphasis of the interesting, de-emphasis of the pedestrian.
- Periodic review and recollection, newly-possible comments about earlier matters.
- Organization at various scales, interactions therebetween.
- Summary.

Procedurally:

- Modern typesetting technology increases our flexibility immensely. Develop the habit of creating a macro as soon as you type something complicated for just the *second* time. The longer you wait, the more work it will be to go back and change all the literals to macro-calls. Refer to objects by label rather than by literal. And so on.

- Rewrite, rewrite, rewrite.

The basic writing unit in Math 111–112 and 211–212 is the exercise-solution.

An exercise-solution is generally one or several paragraphs long.

The adage about after-dinner speaking is:

- Tell them what you're going to tell them.
- Tell them that you're telling it to them.
- Tell them that you've told it to them.

This is a workable model for the exercise-solution genre.

Tell them what you're going to tell them.

Quote the problem, perhaps in a slightly abbreviated form.

Tell them that you're telling it to them.

Solve the problem, explaining what you are doing as you do it.

Tell them that you've told it to them.

End by explaining why the solution is complete, or even just saying that it is complete.

Writing on a larger scale adds further need for large-scale structure and small-scale texture. *Small-scale texture* means appropriate emphasis of currently-live ideas and de-emphasis of other issues.

(Example from complex analysis this term.)

Your ability to write a compelling **introduction** (or **preface**) is a real measure of your large-scale grasp of the work.

Can you lay out the big ideas, the key details, the enlightening examples, into a compelling storyline?

Similarly for your table of contents. It is your outline.

But it is not writ in stone.

Don't fight *chapter-mitosis* when its time has come.

There are many books and pamphlets on mathematical writing. For instance, the Reed library has pieces by Gillman and Krantz.

I suspect that this genre of writing is rather like the calculus book: Each is a labor of care, with valuable contents. But also, each needs to make a common set of basic points. Thus they are probably all similar but all different.

So, it is worth reading through *one* of them carefully, and from then on glancing through others only to see if they have anything significantly different to say. Over time, you may find a favorite (and I would appreciate hearing about it). But finding the one optimal book on mathematical writing is not so important. There is a natural tendency to bond with that in which one invests rewarding work.

In any case, one mostly learns to write mathematics by reading mathematics, not by reading how to write mathematics.

A valuable skill is learning from negative examples.

Don't just say, "This is wretched. I must never do this."

Do the work of formulating *how* it is wretched.

And then, rather than endlessly trying to guard against it, cultivate *positive* habits to prevent its possibility from arising.

The skill of learning positive things from negative circumstances is of great value.

Many Reed alumni return to visit — especially from Ph.D programs — and complain that we did not prepare them for various forms of nastiness:

- Pettiness and harassment.
- Discouragement of curiosity.
- Resentment and idea-theft.
- Indifference and neglect.
- The science-sweatshop.

Again, the trick is to analyze situations rather than just react to them, and then to behave constructively in your own self-interest.

Consider two students who you may have encountered in your tutoring:

- One treats you resentfully as an authority figure who is making them jump through arbitrary hoops.

- One seems genuinely interested in learning what you are trying to help them understand.

Which of them would you recommend as a future tutor?

The way to get into any club is to behave as though one is already a junior member. Show the people involved that you can think from their perspective about their concerns.

Empathy is also a supremely self-serving trait.

Systems need people just as much as people want positions within systems.