MATHEMATICS 332: ALGEBRA — ASSIGNMENT C–H

Reading: Cayley–Hamilton handout.

Problems:

1. Let A be a commutative ring with 1. Let M and N be commutative A-algebras. Explain how to give the tensor product $M \otimes_A N$ the structure of a commutative A-algebra. What issues need to be checked before we know that the process is sensible?

2. Using the notation of the handout, there is an isomorphism

$$\bigwedge\nolimits_{k[x]}^{d} M = \bigwedge\nolimits_{k[x]}^{d} (k[x] \otimes_k V) \approx k[x] \otimes_k \bigwedge\nolimits_{k}^{d} V.$$

How does the dth exterior power of the action of the ring $R = k[x] \otimes_k k[T]$ on M correspondingly transform into an action of R on $k[x] \otimes_k \bigwedge_k^d V$?

3. The handout argues rather quickly that y^{adg} commutes with all of $R^{\wedge 1}$. Supply details as necessary.

4. The handout's display

$$f(x) \otimes 1_V + I = \sum a_i (x \otimes 1_V)^i + I = \sum a_i (1 \otimes T)^i + I = 1 \otimes f(T) + I$$

is missing at least two steps. Provide them.

5. Explain the isomorphisms in the display

$$M/IM = (k[x] \otimes_k V)/(x \otimes 1_V - 1 \otimes T)(k[x] \otimes_k V) \approx 1 \otimes_k V \approx V.$$

6. Look up the coordinate-version of the tensor product of matrices. Then, for

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

write the 4-by-4 matix $A \otimes I_2 - I_2 \otimes A$.