CALCULUS AND ANALYSIS IN EUCLIDEAN SPACE: ADDITIONS AND CORRECTIONS SECOND PRINTING

November 14, 2022

As of autumn 2019, the second printing is available. To check whether a copy of the book is the second printing, see if it has the first few additions and corrections to the first printing.

Chapter 2

- Page 53: A variant proof of Proposition 2.4.7 is as follows: "Let $\{x_\nu\}$ converge to p. Note that $\{x_\nu\}$ is the translate of the null sequence $\{y_\nu\} = \{x_\nu - p\}$ by p. The triangle inequality shows that every translate of a bounded sequence is again bounded, and so it suffices to show that every null sequence $\{y_\nu\}$ is bounded. Given such a sequence, there exists a starting index ν_0 such that $|y_\nu| < 1$ for all $\nu > \nu_0$. For any real number $R > \max\{|y_1|, \dots, |y_{\nu_0}|, 1\}$, we have $\{y_\nu\} \subset B(\mathbf{0}, R)$ as a set. Thus $\{y_\nu\}$ is bounded, as desired."
- Page 57: Near the middle of the page, change "To define connectedness accurately" to "To define connectedness cogently". Toward the bottom of the page, change "see Exercise 2.4.10." to "see Exercise 2.4.10(a). For a definition of connectedness that is a bit subtle but is also readily shown to be a topological property, see Exercise 2.4.10(b)."
- Page 58, Exercise 2.4.10: Make the current exercise be part (a), and add part (b) as follows: "(b) A subset A of $\mathbb{R}^n$ is called **connected** if for every continuous function $f : A \longrightarrow \mathbb{R}$ whose output set lies in the two-element set $\{0, 1\}$, in fact its output set is only $\{0\}$ or $\{1\}$ (or $\varnothing$ if A is empty). The idea here is that A doesn't decompose into two components, because if it did then we could continously map A to $\mathbb{R}$ in a way that takes one component to 0 and the other to 1. Prove that connectedness is a topological property."

Chapter 4

- Page 148: The conditions for the normalized proof of the chain rule should include $g(f(a)) = \mathbf{0}_\ell$ along with $a = \mathbf{0}_n$ and $f(a) = \mathbf{0}_m$. This change needs to be made twice on this page.
- Page 149: The proof of Lemma 4.4.4(1) is more tidily concluded by noting that hk is a twofold product of linear functions and therefore is $o(h, k)$ by Proposition 4.2.6.
- Page 155, line 9: Change "$f_i a)$" to "$f_i(a)$".
- Page 165, beginning of five-line display: Change "$\dfrac{\mathrm{d}F(x,y)}{\mathrm{d}x}$" to "$\dfrac{\mathrm{d}F(x)}{\mathrm{d}x}$".
- Page 173: In the last sentence of exercise 4.6.3(b), change F to $F(x + ct)$ twice and change G to $G(x - ct)$ twice.

- Page 192: After display (4.3), add "(Here we view $\gamma'(t)$ and $\nabla f(\gamma(t))$ as shapeless vectors, because strictly speaking the first is a column and the second a row.)"

Chapter 5

- Page 212: In exercise 5.2.8(a), change the prompt to, "(Because this is a one-dimensional problem, you may verify the old definition of derivative rather than the new one; alternatively, note that $f = T + g$ where T is linear and $g(h)$ is $o(h)$.)"

Chapter 6

- Page 280: A tider proof of Theorem 6.4.2 is, "Define $F_2 : [a,b] \longrightarrow \mathbb{R}$ by $F_2(x) = \int_a^x F'$. Then $F_2' = F'$ by the preceding theorem, so (Exercise 6.4.3) there exists a constant c such that $F_2(x) = F(x) + c$ for all $x \in [a,b]$. Thus, because $F_2(a) = 0$,
$$\int_a^b F' = F_2(b) - F_2(a) = F(b) + c - F(a) - c = F(b) - F(a).\text{"}$$
- Page 282: Replace the paragraph after Corollary 6.4.4 by, "The formula in the corollary is the formula for **integration by inverse substitution**. To obtain it from (6.4), substitute $f \cdot (\phi^{-1})'$ for f in (6.4) to get
$$\int_a^b (f \circ \phi) \cdot ((\phi^{-1})' \circ \phi) \cdot \phi' = \int_{\phi(a)}^{\phi(b)} f \cdot (\phi^{-1})'.$$
By the chain rule, $((\phi^{-1})' \circ \phi) \cdot \phi' = (\phi^{-1} \circ \phi)' = 1$, and so we have the result."
- Page 288: In Figure 6.17, at least three of the gray boxes should be white.
- Page 292, end of first paragraph: The box should be $[-1,1] \times [-1,1]$.
- Page 313, fourth display: "$R_{\geq 0}$" should be "$\mathbb{R}_{\geq 0}$".
- Page 322: Add part (d) to exercise 6.7.15 as follows. "(d) Use various exercises from this section to show that the centroid of the upper half n-ball has last coordinate
$$\overline{x}_n = \frac{2^{n+1}}{\pi(n+1)} \cdot \frac{(n/2)!^2}{n!},$$
and then use Stirling's formula to show that asymptotically, as n grows,
$$\overline{x}_n \sim \sqrt{\frac{2}{\pi n}}.$$
Thus the n-ball is concentrated ever more toward its center."
- Page 324, third display: The lower limit of integration should be $\xi = -\infty$.
- Section 6.8: This section can be omitted if we strengthen the hypotheses of the change of variable theorem to assume that $\Phi(K)$ has boundary of volume zero. For all 2-dimensional and 3-dimensional examples in a calculus class, one can see this to be the case.
- Page 328, line (−3): Change "that that" to "that".

Chapter 9

- Page 443, line (−4): Change "consisted only in" to "consisted only of".
- Page 454, line 98: Replace "recognizing the definition of d" with "product rule, nilpotence of d".
- Page 458: Part (c) of exercise 9.9.3 should begin, "Similarly to part (b), . . . ".

- Page 459: A better end to the proof of Theorem 9.10.1 is

$$\begin{aligned}
\int_\Phi f\,\mathrm{d}x_I &= \int_D (f\circ\Phi)\det\Phi_I' && \text{by definition, as in (9.14)}\\
&= \int_{\Delta^D} (f\circ\Phi)\det\Phi_I'\,\mathrm{d}u_1\wedge\cdots\wedge\mathrm{d}u_k && \text{by Exercise 9.5.4}\\
&= \int_{\Delta^D} \Phi^* f\cdot\Phi^*\mathrm{d}x_I && \text{by Theorem 9.9.3}\\
&= \int_{\Delta^D} \Phi^*(f\,\mathrm{d}x_I) && \text{by Theorem 9.9.4(2).}
\end{aligned}$$

- Page 492: The integral in exercise 9.16.4 is a two-dimensional integral, i.e., $\iint_{\partial\mathbb{H}}$ rather than $\int_{\partial\mathbb{H}}$.
- Page 492: In exercise 9.16.5(b), “closed compact” should just be “compact”.